Sec 4.3 – Recursion Models
Applications of Recursion

Name:

1. Kevin decides to take a job with a company that sells magazine subscriptions. He is paid $20 for the day and then earns $1.50 for each subscription he sells for the day.
   a. Create a table of values showing how much he makes for each number of magazines.

Number of subscription sold	0	1	2	3	4
Amount of pay for the day	$20	$21.50	$23.00	$24.50	$26.00
		20+1(1.50)	20+2(1.50)	20+3(1.50)	20+4(1.50)

   b. Write a **recursive** function for the amount of money Kevin can earn selling magazines.

$$t_{n+1} = t_n + 1.50$$

   c. Write an **explicit** function rule for the n^th term in the sequence describing the amount of money Kevin can earn. Describe any domain restrictions in your rule. How is this rule related to the rule you wrote in the previous question?

$$t_n = 20 + 1.50n$$

1ST TERM JUST USE "n" B/C WE STARTED AT t_0

   d. How much does Kevin earn if he sells 100 magazine subscriptions? Which rule did you use to answer this question? Why?

$$t_{100} = 20 + 1.50(100) = \$170$$

THE EXPLICIT FORMULA IS MORE EFFICIENT

   e. Kevin is trying to earn enough money to buy a new smart phone. He needs $225 to cover the cost and tax on the phone. How many magazine subscriptions does Kevin need to sell to buy his new smart phone?

$$225 = \cancel{20} + 1.5n$$
$$-20 \quad -\cancel{20}$$
$$\frac{205}{1.5} = \frac{1.5n}{1.5}$$
$$136.\overline{6} = n$$

KEVIN NEEDS TO SELL 137 MAGAZINE SUBSCRIPTIONS.

2. Your phone service allows you to add international long distance to your phone. The cost is a $5 flat fee each month and 3¢ a minute for calls made. Write a recursive rule describing your monthly cost for international calls. Then, write a function rule for the n minutes of calls made in a month.

RECURSIVE: $t_{n+1} = t_n + .03$

EXPLICIT: $t_n = 5 + .03n$

3. Alex started a business making bracelets. She sold 30 bracelets the first month. Her goal is to sell 6 more bracelets each month than she sold the previous month. If Alex meets her goal, what is the total number of bracelets she will sell in the first 12 months? How could you go about figuring this out? Can you find a formula?

30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96

126 126 126

$t_{n+1} = t_n + 6$ $6(126) = 756$ BRACELETS   ARITH SERIES: $\frac{n}{2}(a_1 + a_n)$

4. A person was studying a population of a city and noticed that the city's population was growing a consistent rate of 20% per year and could be modeled by the recursive definition $P_{n+1} = 1.20 \cdot P_n$.
What is the growth factor? 1.20

If the population is 22,400 this year, what would the population for each of the next 3 years (assuming the population continues to follow the suggested model)?

$P_0 = 22{,}400$

$P_1 = 1.2(22400) = 26880$

$P_2 = 1.2(26880) = 32256$

$P_3 = 1.2(32256)$

$\approx 38{,}707$

```
22400
1.2*Ans
                22400
                26880
                32256
              38707.2
```

5. A student is playing with pennies to model population growth. The student starts with 2 pennies and flips them at the same time. Another coin is added to the pile of coins for every coin that lands head up (coin flip simulator http://www.shodor.org/interactivate/activities/Coin/).

- What should the growth factor be in theory? INCREASING BY 50% MEANS GROWTH FACTOR 1.5
- Create a table of values showing the number of pennies after each flip?

		H, T	H, T, H	TT HHH	HTTTTTHT
Flip Number	**0**	**1**	**2**	**3**	**4**
Number of Pennies	**2**	3	5	8	10
		$2(1.5)^1$	$2(1.5)^2$	$2(1.5)^3$	$2(1.5)^4$

- Create a **recursive** definition.

$t_{n+1} = 1.5 t_n$

- Create an **explicit** definition.

$t_n = 2 \cdot 1.5^{\wedge}$

6. A school attendance clerk noticed that the first week after winter break 5 students were out with the flu. By the end of the second week, a total of 9 students had come down with the flu at one time and by the third week 16 had the flu at one time. The attendance clerk noticed the number of students that had the flu was growing exponentially with a growth factor of 1.8.

- Create a table of values showing the number of pennies after each flip?

Week	**1**	**2**	**3**	**4**	**….**
Number of Students that have had the flu	**5**	**9**	**16**	**29**	**….**
	$5(1.8)^0$	$5(1.8)^1$	$5(1.8)^2$	$5(1.8)^3$	

- Create a **recursive** definition.

$t_{n+1} = 1.8 t_n$

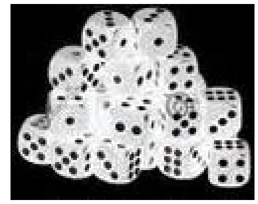
- Create a **explicit** definition.

$t_n = 5 \cdot (1.8)^{n-1}$

- If the model continues how many would have had the flu by the 8th week of school?

$t_8 = 5 \cdot (1.8)^7 \approx \boxed{306 \text{ STUDENTS}}$

5. A student is playing with dice to simulate a declining population. They started with **50** dice and any dice that land on an ‘ODD’ number are removed. Then, the remaining dice are rolled and again, the dice that land on an ‘ODD’ number are removed. The process is repeated until no dice are left. (dice roll simulator https://www.random.org/dice/).



- What should the ~~growth~~ decay factor be in theory? THE PROBABILITY OF AN ODD NUMBER IS 50%

  $100\% - 50\% = 50\%$ OR 0.50

- Create a table of values showing the number of pennies after each flip?

Roll Number	0	1	2	3	…	??
Number of Pennies	50	≈ 25	≈ 13	≈ 6	…	≈ 0

$50(0.5)^0$ $50(0.5)^1$ $50(0.5)^2$ $50(0.5)^3$ 7TH

$50(0.5)^7 \approx 0$

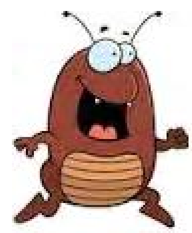
- Create a **recursive** definition.

  $t_{n+1} = 0.5\, t_n$

- Create an **explicit** definition.

  $t_n = 50\,(0.5)^n$

6. A population starts with 16 fleas and in a day the population increases by 3 new fleas for every 4 fleas the previous day which can be shown in the following table



Day	0	1	2	3	4
Number of Fleas	16	28	49	≈ 86	≈ 150

$16(1.75)^0$ $16(1.75)^1$ $16(1.75)^2$ $16(1.75)^3$ $16(1.75)^4$

- What is the growth factor? THE POPULATION IS INCREASING BY $3/4 = 0.75 = 75\%$ SO THE GROWTH FACTOR $100\% + 75\% = 175\% = 1.75$.

- Create a **recursive** definition.

  $t_{n+1} = 1.75 \cdot t_n$

- Create an **explicit** definition.

  $t_n = 16 \cdot (1.75)^n$

- If the model continues how many fleas would you predict there will be on day 9?

  $t_9 = 16\,(1.75)^9 \approx \boxed{\text{2463 FLEAS}}$

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16(1.75)^9
        2462.988708
```

7. Trisha decided to start an exercise program in which on the first day she did 4 sit-ups. The next day she would do 2 more sit-ups than the previous day. She wanted to continue doing 2 more than the previous day for 10 days.

Day	~~1~~ 0	~~2~~ 1	~~3~~ 2	~~4~~ 3	~~5~~ 4	….
Sit-Ups for the day	**4**	**6** 4+1(2)	**8** 4+2(2)	**10** 4+3(2)	**12** 4+4(2)	….
Total number of Sit-Ups	**4**	**10**	**18**	**28**	**40**	….

- What type of sequence does the number of sit-ups each day create? ARITHMETIC SEQUENCE

4, 6, 8, 10, 12, 14, 16, 18, 20, 22

26 26 26 26 26

- How many sit-ups will she have done by the end of the 10 days?

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4+6+8+10+12+14+1
6+18+20+22
                 130
5(26)
                 130
```

$5\cdot(26) = 130$ SIT UPS

5 PAIRS OF NUMBERS THAT SUM TO 26

- Could you use your technique to find how many sit-ups she would have done if she did this for 20 days?

$t_n = 4 + (n-1)2$

$t_{20} = 4 + (19)2$

$t_{20} = 42$

4, 6, 8, 10, ……, 38, 40, 42 ← DAY 20

46 46 46

$10(46) = 460$ SIT UPS

• WITH 20 DAYS THAT WOULD MAKE 10 PAIRS OF 46

8. Jason started a new job in which the employer required that he put $120 into a retirement fund each month. The company would also add 1% of the amount in his retirement fund each month.

Month	**m_1**	**m_2**	**m_3**	**m_4**	**m_5**	**m_6**
Account Balance	**0**	**\$120**	**\$241.20**	**\$363.61**	\$487.25	\$612.37
1% interest earned	**0**	**\$1.20**	**\$2.41**	**\$3.64**	\$4.87	\$6.12
Money added	**\$120**	**\$120**	**\$120**	**\$120**	+ \$120	+ \$120
	+ 120	+ 241.20	+ 363.61	+ 487.25	\$612.37	\$738.49

- How much money will be in his retirement account at the end of 6 months?

\$738.49