Sec 4.1 – Recursion Models
Recursive & Explicit

Name:

Complete the next 3 terms in the sequence and give a RECURSIVE DEFINITION (i.e. a rule on how to go from one term to the next)

1. 2,4,6,8, 10 , 12 , 14 RULE: $t_{n+1} = t_n + 2$

2. 2,4,8,16,32 64 , 128 , 256 RULE: $t_{n+1} = 2 \cdot t_n$

3. 4, 12, 36, 108, 324 , 972 , 2916 RULE: $t_{n+1} = 3 \cdot t_n$

4. 8, 5, 2, -1, -4 , -7 , -10 RULE: $t_{n+1} = t_n - 3$

5. 54, 18, 6, 2, $\frac{2}{3}$, $\frac{2}{9}$, $\frac{2}{27}$ RULE: $t_{n+1} = t_n \div 3$

6. 4, 10, 16, 22, 28 , 34 , 40 RULE: $t_{n+1} = t_n + 6$

7. 2, 7, 22, 67, 202 , 607 , 1822 RULE: $t_{n+1} = 3 \cdot t_n + 1$

ONE TWO THREE FOUR FIVE SIX SEVEN
8. O, T, T, F, F,S, S, E , N , T , E , T

9. J , A, S, O, N, D , J , F , M , A

10. 1, 4, 9, 61, 52, 63, 94, 46 , 18 , 001

11. A , , , , E , F , , H , I , , K
B C D G J

12. A , B , , D , , , , , , , ,
C E F G H I J K L M N O

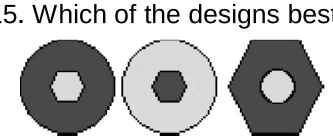
13.
1
1 1
2 1
1 2 1 1
1 1 1 2 2 1
3 1 2 2 1 1
1 3 1 1 2 2 2 1

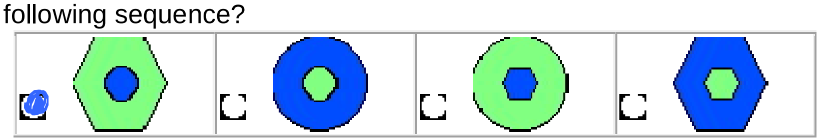
14. Each number shown below follows a certain rule. Figure out the rule and fill in the missing number.

January	20
April	10
May	5
November	15
July	? 10

SYLLABLES • 5

15. Which of the designs best completes the following sequence?



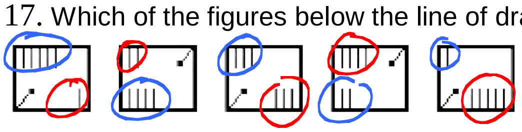


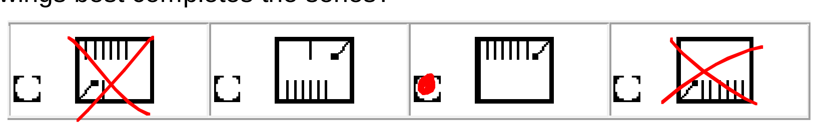
16. Which letter comes next in this series of letters?

B A C B D C E D F ? G

- [ ] C
- [ ] D
- [x] E
- [ ] F
- [ ] G

17. Which of the figures below the line of drawings best completes the series?





Recursive functions vs. Explicit functions

Create a sequence from the following.
Recursive Definition:
$t_{n+1} = t_n + 4$ and $t_1 = 3$

$n=1: t_2 = t_1 + 4 = 3+4 = 7$
$n=2: t_3 = t_2 + 4 = 7+4 = 11$

$t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5 \quad t_6$
$3, 7, 11, 15, 19, 23, \ldots$

Find t_{10} $= 39$

Create a sequence from the following.
Explicit Definition:
$t_n = 4n - 1$

$n=1: t_1 = 4(1) - 1 = 3$
$n=2: t_2 = 4(2) - 1 = 7$
$n=3: t_3 = 4(3) - 1 = 11$
$n=4: t_4 = 4(4) - 1 = 15$

$t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5$
$3, 7, 11, 15, 19, \ldots$

Find t_{10} : $n=10: t_{10} = 4(10) - 1 = 40 - 1 = 39$

How would you define the following?
Recursive Definition – a mathematical function that describes future terms of a sequence based on previous terms.

Explicit Definition – a mathematical function that describes any term of the sequence given the term number.

18. Create the first few terms of a sequence using the following ***Recursive Definitions:***

a. $t_{n+1} = 2 \cdot t_n - 1$ and $t_1 = 2$

$2, 3, 5, 9, 17, \ldots$
$t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5$

$n=1: t_2 = 2 \cdot t_1 - 1 = 2(2) - 1 = 3$
$n=2: t_3 = 2 \cdot t_2 - 1 = 2(3) - 1 = 5$
$n=3: t_4 = 2 \cdot t_3 - 1 = 2(5) - 1 = 9$
$n=4: t_5 = 2 \cdot t_4 - 1 = 2(9) - 1 = 17$

b. $a_{n+1} = 2^{(a_n)} - 1$ and $a_1 = 2$

$2, 3, 7, 127, \ldots$
$a_1 \quad a_2 \quad a_3 \quad a_4$

$n=1: a_2 = 2^{a_1} - 1 = 2^2 - 1 = 3$
$n=2: a_3 = 2^{a_2} - 1 = 2^3 - 1 = 7$
$n=3: a_4 = 2^{a_3} - 1 = 2^7 - 1 = 127$

c. $t_n = (t_{n-1})^2 - 4$ and $t_1 = 3$

$3, 5, 21, 437, \ldots$
$t_1 \quad t_2 \quad t_3 \quad t_4$

$n=1: \cancel{t_1 = (t_0)^2 - 4}$
$n=2: t_2 = (t_1)^2 - 4 = (3)^2 - 4 = 5$
$n=3: t_3 = (t_2)^2 - 4 = (5)^2 - 4 = 21$
$n=4: t_4 = (t_3)^2 - 4 = (21)^2 - 4 = 437$

19. Complete the table.

$a_n = 3 \cdot a_{n-2} - a_{n-1} - 1$

n	**1**	**2**	**3**	**4**	**5**	**6**
$\mathbf{a_n}$	**3**	**4**	**4**	**7**	4	16
	a_1	a_2	a_3	a_4	a_5	

$n=5$: $a_5 = 3a_3 - a_4 - 1$
$= 3(4) - (7) - 1$
$= 12 - 7 - 1$
$= 4$

$n=6$: $a_6 = 3 \cdot a_4 - a_5 - 1$
$= 3(7) - (4) - 1$
$= 21 - 4 - 1$
$= 16$

20. Generate 3 different sequences that could be defined by $t_{n+1} = 2 \cdot t_n + 1$

$t_1 = 1$: $1, \underset{2(1)+1}{3}, \underset{2(3)+1}{7}, \underset{2(7)+1}{15}, \ldots$

$t_1 = 4$: $4, \underset{2(4)+1}{9}, \underset{2(9)+1}{19}, \underset{2(19)+1}{39}, \ldots$

$t_1 = 5$: $5, \underset{2(5)+1}{11}, \underset{2(11)+1}{23}, \underset{2(23)+1}{47}, \ldots$

21. Create the first few terms of a sequence using the following ***Explicit Definitions:***

a. $t_n = 2 \cdot n - 1$

$\underset{t_1}{1}, \underset{t_2}{3}, \underset{t_3}{5}, \underset{t_4}{7}, \ldots$

$n=1$: $t_1 = 2(1) - 1 = 1$

$n=2$: $t_2 = 2(2) - 1 = 3$

$n=3$: $t_3 = 2(3) - 1 = 5$

$n=4$: $t_4 = 2(4) - 1 = 7$

b. $a_n = 3^n - n$

$\underset{a_1}{2}, \underset{a_2}{7}, \underset{a_3}{24}, \underset{a_4}{77}, \ldots$

$n=1$: $a_1 = 3^1 - 1 = 2$

$n=2$: $a_2 = 3^2 - 2 = 7$

$n=3$: $a_3 = 3^3 - 3 = 24$

$n=4$: $a_4 = 3^4 - 4 = 77$

c. $t_n = 5 \cdot n - 4$ and determine t_{23}

$n=23$: $t_{23} = 5(23) - 4$
$= \boxed{111}$

22. Create the first few terms of a sequence using the following **mixed** ***Recursive Definitions:***

a. $a_{n+1} = n + 2 \cdot a_n - 1$ and $a_1 = 3$

$3, 6, 13, 28, \ldots$

$a_1 \quad a_2 \quad a_3 \quad a_4$

$n=1: \; a_2 = (1) + 2a_1 - 1$
$= 1 + 2(3) - 1$
$= 1 + 6 - 1 = \underline{6}$

$n=2: \; a_3 = (2) + 2a_2 - 1$
$= 2 + 2(6) - 1$
$= 2 + 12 - 1 = 13$

$n=3: \; a_4 = (3) + 2a_3 - 1$
$= 3 + 2(13) - 1$
$= 3 + 26 - 1 = 28$

b. $t_n = 4 \cdot t_{n-2} - t_{n-1} - n$ and

n	1	2	3	4	5
t_n	3	4	5	7	8
	t_1	t_2	t_3	t_4	

$n=5: \; t_5 = 4 \cdot t_3 - t_4 - (5)$
$= 4(5) - (7) - 5$
$= 20 - 7 - 5 = 8$